On the neutrality issue in the Polyakov-loop NJL model

H. Abuki,[1, *] M. Ciminale,[2, 3, †] R. Gatto,[4, ‡] and M. Ruggieri[2, 3, §]

[1]*Institut für Theoretische Physik, J.W. Goethe Universität, D-60438 Frankfurt am Main, Germany*
[2]*I.N.F.N., Sezione di Bari, I-70126 Bari, Italy*
[3]*Università di Bari, I-70126 Bari, Italy*
[4]*Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland*
(Dated: February 7, 2009)

We elucidate how the color neutrality is harmed in the Polyakov Nambu-Jona Lasinio (PNJL) model at finite density within the adopted mean field approximation. Also we point out how usual assumption about the diagonal form of the Wilson loop may fail in the presence of the diquark condensate on several grounds.

I. INTRODUCTION

In recent years several aspects of the Quantum Chromodynamics (QCD) phase diagram in the $\mu - T$ plane have been investigated. Here μ denotes the quark chemical potential and T corresponds to the temperature. The studies are mainly related to the breaking and/or restoring of the global symmetries of the QCD lagrangian in the ground state. The high temperature-small chemical potential region of the phase diagram can be accessed by lattice calculations, see for example[1, 2, 3, 4] and reference therein, as well as effective models, among others chiral perturbation theory (ChPT) [5, 6], quark meson coupling (QMC) [7, 8] and the Nambu-Jona Lasinio model(NJL) [9, 10] . On the other hand the dual region of low temperature-large chemical potential is not accessible nowadays by lattice calculations because of the well known sign problem of 3-color QCD (for reviews see[11, 12, 13]). In this case our knowledge of the QCD phase diagram is mainly based on phenomenological models at intermediate density and perturbative QCD at very high density.

One of the most interesting features of the QCD phase diagram at high temperature and low density is the chiral crossover. The QCD chiral symmetry is explicitly broken by the bare quark masses and spontaneously broken by chiral condensates[14]. In the vacuum, i.e. at $\mu = T = 0$, the chiral condensate $\langle \bar\psi\psi \rangle$ is expected to be of the order of $-(250 \text{ MeV})^3$. Lattice calculations show that there exists a temperature of the order of 200 MeV at which the chiral condensate smoothly drops down to a lower value, and at the same time chiral susceptibilities develop a peak, signaling the existence of a smooth crossover. The Goldstone bosons related to the chiral symmetry breaking are the pions, which are massive below the phase transition temperature because of the bare quark masses. Their mass increases above the critical temperature so that they decouple from the low energy spectrum, and this can be interpreted as a further signal of the restoration of the chiral symmetry. The smooth crossover becomes a true phase transition when one artificially sets the bare quark masses to zero, so that the chiral symmetry becomes an exact symmetry of the QCD lagrangian, and above the critical temperature the chiral condensate is zero.

Lattice calculations show that in the neighborhood of the chiral crossover a Polyakov loop crossovers occurs[15, 16, 17]. The Polyakov loop Φ is an order parameter for deconfinement in the pure gauge theory[18, 19, 20]. In more detail, the expectation value of the Polyakov loop $\langle \Phi \rangle$ is related to the free energy F_q of a heavy (thus non dynamical) quark by the relation $\langle \Phi \rangle \propto \exp(-\beta F_q)$. The calculations of $\langle \Phi \rangle$ as a function of the temperature in lattice-QCD (LQCD) give the result that $\langle \Phi \rangle = 0$ for $T < T_0$ and $\langle \Phi \rangle \neq 0$ at $T > T_0$. Here T_0 denotes the so called deconfinement temperature of the pure glue theory, $T_0 \approx 270$ MeV. The LQCD results imply that for $T < T_0$ the free energy of a static quark is infinite and therefore it can not be produced. On the other hand for $T > T_0$ the expectation value of the Polyakov loop is no longer zero, thus the free energy of a static quark is finite and it can be produced. Beside this the correlation of a Polyakov loop and a Polyakov antiloop at finite distance, namely $\langle \bar\Phi(0)\bar\Phi(x) \rangle$, is related to the interaction energy of a heavy quark and a heavy antiquark. At $T < T_0$ the correlation grows linearly with the distance x, thus enforcing the confining feature of the low temperature region, while at $T > T_0$ it becomes a screened Coulomb potential. The Polyakov loop is a gauge singlet but it is not invariant under an element of the Z_3 group. Therefore in

the deconfined phase the Z_3 symmetry is spontaneously broken while in the confined phase it is unbroken. For this reason the Polyakov loop is often referred to as the order parameter of the confinement-deconfinement transition.

In presence of dynamical quarks one can no longer regard Φ as an order parameter. The very reason for this is that dynamical quarks break explicitly the Z_3 symmetry and thus one expects $\langle \Phi \rangle \neq 0$ even in the previously called confined phase. Nevertheless the Polyakov loop is supposed to serve as an indicator of the phase transition, since LQCD results show that a Polyakov loop crossover occurs almost in coincidence with the chiral crossover at high temperature. The coincidence of the two crossovers suggests that chiral symmetry restoration and deconfinement are intimately related.

These results are confirmed by effective model calculations. Among the numerous effective models the most celebrated one is certainly the NJL model (for reviews see [21, 22, 23]). In the NJL model one replaces the gluon-mediated quark interaction by a 4-fermion interaction, and the chiral condensate is computed by solving a non linear gap equation mimicking the BCS effective model of ordinary superconductors[24]. Once the parameters of the model are adjusted in order to reproduce some phenomenological vacuum properties of QCD (meson masses, chiral condensate and pion decay constant) the model can be used to predict the existence of a chiral crossover at a temperature of the order of 200 MeV, in agreement with LQCD. Unfortunately the original version of the NJL model does not incorporate gluons as dynamical degrees of freedom. As a consequence the interplay between the chiral and the Polyakov loop crossover can not be discussed within the model. This obstacle has been recently surmounted by the NJL model with the Polyakov loop (PNJL model in the following)[25, 26].

In the PNJL model one introduces an effective potential $\mathcal{U}$ for the Polyakov loop and its conjugate. Its parameters are tuned to reproduce the LQCD results with no dynamical quarks, namely the deconfinement temperature T_0 and the thermodynamics of the pure glue liquid for temperatures close to T_0. Moreover assuming the existence of a background temporal gluon field, the coupling of the Polyakov loop to the dynamical quarks is achieved via the QCD coupling of the background field and the quarks themselves. For more formal details see next Section. The PNJL model successfully reproduces several LQCD results at zero and small quark chemical potential, as well as results with imaginary chemical potential.

We have demonstrated in [27], that the Polyakov Nambu-Jona Lasinio (PNJL) model [25] *only* at nonzero baryon chemical potential may have some intrinsic problem of color neutrality. As stressed there, this issue does not exist at zero density, in no contradiction with the zero density results in [28]. However, once we introduce the baryon chemical potential, any PNJL-type model is no longer free from the color neutrality problem. This may be viewed as an artifact of the PNJL model, or possibly of the mean-field approximation scheme usually taken in the literature. In this article we wish to clarify how this problem can emerge at finite density, irrespective to the existence of diquark fields.

II. FORMAL SETTINGS

First we illustrate the problem at finite density but without the diquark condensate. To be confident, we here start with the NJL model for two-flavor quarks, combined with a matrix model in order to derive the PNJL type model [29]. Apart from the gluon part, the partition function of the PNJL model is given as

$$Z[\mu, T] \;=\; \int \Pi_{\boldsymbol{x}} dL(\boldsymbol{x}) \int dq d\bar{q} e^{-\int_0^{1/T} d\tau \int dx \mathcal{L}_E(q, \bar{q}; A_4)}, \tag{1}$$

with the Euclidean lagrangian density defined as

$$\mathcal{L}_E(q, \bar{q}, A_4) = q^+ (\gamma_0(\partial_\tau + iA_4(x)) + i\boldsymbol{\gamma} \cdot \nabla + m - \not{\mu})q - G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2 \right]. \tag{2}$$

L is the Wilson-loop matrix, $L \in$ SU(3). In the hypothesis that the system is well-described by the mean-field approximation at large distance [30], we replace the above group integral over the all space with a single integral over L, i.e., we take $L(\boldsymbol{x})$ to be a constant over the whole space, which is an implicit assumption also in the PNJL model [25].

$$L(\boldsymbol{x}) = P_\tau e^{i\int_0^{1/T} d\tau A_4(x)} \stackrel{\text{PNJL}}{\rightarrow} L = e^{iA_4/T}, \tag{3}$$

where $A_4 = \sum_{\alpha=1}^8 \phi_\alpha \lambda_\alpha$ with $\{\phi_\alpha\}$ the real parameters. By introducing the auxiliary field $\sigma \sim 2G\bar{q}q$, and then integrating out the quark fields we obtain

$$Z[\mu, T] = \int dL \int_{\text{P.B.}} \mathcal{D}\sigma(x) e^{-S_{\text{eff}}[A_4; \sigma(x)\mu, T]}, \tag{4}$$

where the integral over L is replaced by the global one. The effective action in the presence of constant A_4 background is thus

$$S_{\text{eff}}[A_4, \sigma(x); \mu, T] = \int_0^{1/T} d\tau \int \frac{\sigma(x)^2}{4G} - \log \text{Det}_{x,y} \left[(-i\not{D}_x[\mu, A_4] + m - \sigma(x))\delta_{\text{P}}(x - y) \right], \tag{5}$$

where the covariant derivative is defined by $i\not{D}[\mu, A_4] \equiv \gamma_0(\partial_\tau + iA_4 - \mu) - i\boldsymbol{\gamma} \cdot \nabla_x$, and $\delta_{\text{P}}(x - y)$ is the delta function periodic in imaginary time τ. In the above equation we have assumed that in the ground state pion condensation does not take place, resulting in the vanishing of the pseudoscalar condensate $\langle \bar{q}\gamma_5 \boldsymbol{\tau} q \rangle$. This simplification is justified by our previous results that show that in the physical limit pion condensation is forbidden in the ground state of the model [31, 32, 33].

We look for the homogeneous ground state via a saddle-point approximation. In this case, the approximated action is space-time independent and equal to $\beta V \Omega[A_4, \sigma; \mu, T]$, with thermodynamic potential Ω given by

$$\Omega(L, \sigma; \mu, T) = \frac{\sigma^2}{4G} - 2N_{\text{f}}T \int \frac{d\boldsymbol{p}}{(2\pi)^3} \text{tr}_c \ln \left(1 + L^+ e^{-(E-\mu)/T} \right) - 2N_{\text{f}}T \int \frac{d\boldsymbol{p}}{(2\pi)^3} \text{tr}_c \ln \left(1 + L e^{-(E+\mu)/T} \right), \tag{6}$$

where $E = \sqrt{p^2 + M^2}$ with $M = m - \sigma$. As usual, the vacuum part of the above integral is divergent, so we need to put the sharp cutoff Λ in the momentum integral. We should note that in contrast to the $\mu = 0$ case, Ω is not symmetric under the the gluonic charge conjugation $L \leftrightarrow L^+$.[1]

Due to the global color SU(3) invariance of the thermodynamic function of the loop, we can always find $g \in$ SU(3) such that $\Omega[L; \mu, T] = \Omega[gL_{\text{diag}}g^{-1}; \mu, T] \equiv \Omega[L_{\text{diag}}; \mu, T]$ where L_{diag} can be parameterized by

$$L_{\text{diag}} = \text{diag}.(e^{i\varphi_1}, e^{i\varphi_2}, e^{-i\varphi_1 - i\varphi_2}). \tag{7}$$

Then integrating out the "phase" g using the property of the Haar measure $dL = \mu(\varphi_1, \varphi_2)d\varphi_1 d\varphi_2[dg]$, the partition function becomes

$$Z[\mu, T] = \int \mu(\varphi_1, \varphi_2)d\varphi_1 d\varphi_2 e^{-\beta V \Omega[L_{\text{diag}}; \mu, T]}, \tag{8}$$

where $\mu(\varphi_1, \varphi_2)$ is the measure for the eigenvalue distribution of SU(3) that is proportional to the squared Vandermonde determinant $[\Delta(e^{i\varphi_1}, e^{i\varphi_2}, e^{-i\varphi_1 - i\varphi_2})]^2$ [34]. The correctly normalized measure becomes

$$\mu(\varphi_1, \varphi_2) = \frac{(\sin(\varphi_1 - \varphi_2) - \sin(2\varphi_1 + \varphi_2) + \sin(\varphi_1 + 2\varphi_2))^2}{6\pi^2}. \tag{9}$$

Here the variables are defined in the interval $\{\varphi_1, \varphi_2\} \in [0, 2\pi]$. Now we introduce the independent variables $\{\Phi, \bar{\Phi}\}$ by $\Phi(\varphi_1, \varphi_2) = \frac{1}{3}\text{tr}_c L_{\text{diag}}$ and its conjugate $\bar{\Phi} = \Phi^*$; we have $\mu(\varphi_1, \varphi_2) = \frac{9}{8\pi^2}(1 - 6\Phi\bar{\Phi} + 4\Phi^3 + 4\bar{\Phi}^3 - 3\Phi^2\bar{\Phi}^2)$. Then we may write down the full PNJL partition function as

$$\begin{aligned} Z_{\text{PNJL}}[\mu, T] &\equiv \int d\varphi_1 d\varphi_2 e^{-S_\varphi[\Phi(\varphi_1,\varphi_2), \bar{\Phi}(\varphi_1,\varphi_2); \mu, T]}, \\ S_\varphi[\Phi, \bar{\Phi}, \sigma; \mu, T] &= \beta V \left(\Omega[\Phi, \bar{\Phi}, \sigma; \mu, T] + V_{\text{glue}}[\Phi, \bar{\Phi}] \right). \end{aligned} \tag{10}$$

In the above equation $\Omega[\Phi, \bar{\Phi}, \sigma; \mu, T] = \Omega[L, \sigma; \mu, T]$ whose explicit form is given by

$$\begin{aligned} \Omega[\Phi, \bar{\Phi}, \sigma; \mu, T] &= \frac{\sigma^2}{4G} - 2N_{\text{f}}T \int \frac{d\boldsymbol{p}}{(2\pi)^3} \ln(1 + 3\bar{\Phi}e^{-(E-\mu)/T} + 3\Phi e^{-2(E-\mu)/T} + e^{-3(E-\mu)/T}) \\ &\quad - 2N_{\text{f}}T \int \frac{d\boldsymbol{p}}{(2\pi)^3} \ln(1 + 3\Phi e^{-(E+\mu)/T} + 3\bar{\Phi}e^{-2(E+\mu)/T} + e^{-3(E+\mu)/T}). \end{aligned} \tag{11}$$

$V_{\text{glue}}[\Phi, \bar{\Phi}]$ is the Polyakov-loop potential in the PNJL model specified below, which includes the phenomenological mass term proportional to $\Phi^*\Phi$, and the potential inspired by the group theoretical constraint, i.e., the above-derived Vandermonde determinant.

$$\frac{1}{T^4} V_{\text{glue}}[\Phi, \bar{\Phi}] = -\frac{b_2(T)}{2}\bar{\Phi}\Phi + b_4(T) \ln(1 - 6\Phi\bar{\Phi} + 4\Phi^3 + 4\bar{\Phi}^3 - 3\Phi^2\bar{\Phi}^2). \tag{12}$$

---

[1] The partition function is symmetric only under the full charge conjugation acting on both the quark and gluon sectors, i.e., $L \to L^+$ together with $\mu \to -\mu$.

$b_2(t)$ and $b_4(T)$ are the phenomenological parameters, and may be set as in [28].

In the next Section we show that the non-vanishing color density appears in the model only at $\mu \neq 0$. The point is that Φ and $\bar{\Phi}$ are the *global* parameters in the PNJL model, and so are $\{\varphi_1, \varphi_2\}$. We should notice that by this simplification we can integrate over the fermions in the partition function. This simplification, however, inevitably introduces an artificial global color symmetry breaking in this model, which may be regarded as an artifact as we discuss below.

A. A comment on the conjugated loop in the PNJL model.

The expectation values for loop and conjugated loop can be determined in principle by [30]

$$
\begin{aligned}
\langle \Phi \rangle_{\mu,T} &= Z_{\text{PNJL}}^{-1} \int d\varphi_1 d\varphi_2 \left[(\text{Re}\,\Phi) e^{-\text{Re}\,S_\varphi} \cos(\text{Im}\,S_\varphi) + (\text{Im}\,\Phi) e^{-\text{Re}\,S_\varphi} \sin(\text{Im}\,S_\varphi) \right], \\
\langle \bar{\Phi} \rangle_{\mu,T} &= Z_{\text{PNJL}}^{-1} \int d\varphi_1 d\varphi_2 \left[(\text{Re}\,\Phi) e^{-\text{Re}\,S_\varphi} \cos(\text{Im}\,S_\varphi) - (\text{Im}\,\Phi) e^{-\text{Re}\,S_\varphi} \sin(\text{Im}\,S_\varphi) \right].
\end{aligned}
\tag{13}
$$

Here we have used the fact $\text{Im}\,\Omega$ is odd with respect to the charge conjugation $\mathcal{C}$ acting on gluon sector, i.e., $\Phi \leftrightarrow \bar{\Phi}$. It is notable that $\langle \Phi \rangle \neq \langle \bar{\Phi} \rangle$ but both stay real [30]. Noting that the contribution to the $\sin(\text{Im}\,S_\varphi)$ term in Z_{PNJL} from L^+ configuration will be completely opposite to that from L, we could also write down the partition function which is manifestly real as

$$
Z_{\text{PNJL}}[\mu, T] = \int d\varphi_1 d\varphi_2 e^{-\text{Re}\,S_\varphi} \cos(\text{Im}\,S_\varphi).
\tag{14}
$$

It is often said that $\langle \Phi \rangle = \langle \bar{\Phi} \rangle$ in the mean field approximation, but this is not true. It is the direct consequence of discarding the imaginary part of the action. In most of the literatures, $\mathcal{C}$-odd piece $\text{Im}\,S_\varphi$ is discarded, which is actually responsible for the equality between the expectation values of loop and antiloop.

III. GLOBAL COLOR SYMMETRY BREAKING IN THE PNJL MODEL

Now we turn to the problem of global color symmetry breaking. We adopt here the widely used simplification to discard the imaginary piece of S_φ. In this way we should have $\langle \Phi \rangle = \langle \bar{\Phi} \rangle$ since the simplified action no longer has the $\mathcal{C}$-odd part. This means that we should use the saddle point approximation with a constraint $\Phi = \bar{\Phi}$. This can be done by taking a constraint $\varphi_1 = -\varphi_2 \equiv \varphi$. This is similar to gauge fixing. Now we take the saddle point approximation, namely we look for the values of Φ and σ that minimize the thermodynamical potential:

$$
\left. \frac{\delta \text{Re}\,S_\varphi}{\delta \varphi} \right|_{\{\varphi = \varphi_0, \sigma = \sigma_0\}} = 0, \quad \left. \frac{\delta \text{Re}\,S_\varphi}{\delta \sigma} \right|_{\{\varphi = \varphi_0, \sigma = \sigma_0\}} = 0,
\tag{15}
$$

where $\text{Re}\,S_\varphi = \beta V(\Omega[\Phi, \Phi, \sigma; \mu, T] + V_{\text{glue}}[\Phi, \Phi])$ with $\Phi = \frac{1 + 2\cos\varphi}{3}$; the above equation defines the mean field values of Φ_0 and σ_0. This is the usual procedure in the PNJL model framework. Apparently this breaks the balance between *red*, *green* quark and *blue* quark since the Wilson loop L is asymmetric in color space within the gauge in which $\varphi_1 = -\varphi_2 \equiv \varphi$. We name this configuration as $L_{\text{diag}}^{(1)}$, that is

$$
L_{\text{diag}}^{(1)} = \begin{pmatrix} e^{i\varphi} & 0 & 0 \\ 0 & e^{-i\varphi} & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\tag{16}
$$

In this configuration we have observed a finite color density $\langle q^+ T_8 q \rangle = n_8 \neq 0$ for $\mu \neq 0$ [27]; the price to pay to avoid this fact is to introduce the color chemical potential μ_8, whose self consistently computed value assures the vanishing of the color charge (see also Ref. [35]). [2] When the diquark condensate comes to play a role, $\langle q^+ T_8 q \rangle = n_8 \neq 0$ at $\mu_8 = 0$ regardless of the presence/absence of the Polyakov loop. One could think that it does not bring any problem because of a residual discrete "gauge" freedom which remains even after a continuous gauge freedom is lost by the gauge fixing $L \to g L_{\text{diag}} g^{-1}$. This remnant is nothing but a permutation of (r, g, b) quarks.

---

[2] At $\mu = 0$, quark density and antiquark density are balanced so that there is no color density in the system even if $\mu_8 = 0$. This is consistent with the results obtained in [28].

One must, of course, keep in mind that several configurations that give the same Φ_0 may contribute to the total $Z^{\mathrm{MF}}_{\mathrm{PNJL}}[\Phi_0, \sigma_0; \mu, T]$ via the integral given in Eq. (10). Let us first introduce explicitly the elements of the permutation group, which is subgroup of SU(3). The followings are the elements of $(1) \Leftrightarrow (r, g\ flip)$, $(2) \Leftrightarrow (g, b\ flip)$, $(3) \Leftrightarrow (b, r\ flip)$.

$$g_{rg} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad g_{gb} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad g_{br} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{17}$$

Using the above discrete transformations, we can produce another configuration (different gauge) which realizes the same value of Φ. We name the (r, g) flipped configuration $L^{(2)}_{\mathrm{diag}}$, that is

$$L^{(2)}_{\mathrm{diag}} = g_{rg} L_{\mathrm{diag}} g_{rg}^{-1} = \begin{pmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & e^{i\varphi} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{18}$$

From these $L^{(1)}_{\mathrm{diag}}$ and $L^{(2)}_{\mathrm{diag}}$, we can construct four more configurations as

$$
L^{(3)}_{\mathrm{diag}} = g_{gb} L^{(1)}_{\mathrm{diag}} g_{gb}^{-1} = \begin{pmatrix} e^{i\varphi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\varphi} \end{pmatrix}, \quad L^{(4)}_{\mathrm{diag}} = g_{gb} L^{(2)}_{\mathrm{diag}} g_{gb}^{-1} = \begin{pmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\varphi} \end{pmatrix},
$$
$$
L^{(5)}_{\mathrm{diag}} = g_{br} L^{(1)}_{\mathrm{diag}} g_{br}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\varphi} & 0 \\ 0 & 0 & e^{i\varphi} \end{pmatrix}, \quad L^{(6)}_{\mathrm{diag}} = g_{br} L^{(2)}_{\mathrm{diag}} g_{br}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\varphi} & 0 \\ 0 & 0 & e^{-i\varphi} \end{pmatrix}. \tag{19}
$$

All these configurations $\{L^{(1)}, L^{(2)}, \cdots, L^{(6)}\}$ give the same Φ. In the original variables (φ_1, φ_2) parametrizing L_{diag} as $L_{\mathrm{diag}} = \mathrm{diag}.(e^{i\varphi_1}, e^{i\varphi_2}, e^{-i\varphi_1 - i\varphi_2})$, these configurations correspond to $\{(\varphi, 2\pi - \varphi)_1, (2\pi - \varphi, \varphi)_2, (\varphi, 0)_3, (2\pi - \varphi, 0)_4, (0, \varphi)_5, (0, 2\pi - \varphi)_6\}$. This will become clearer in the diagonal color SU(3) basis, $\{\lambda_3, \lambda_8\}$, writing $L_{\mathrm{diag}} = e^{i\phi_3 \lambda_3 + i\phi_8 \lambda_8}$. In this base, these configurations make a hexahedron in the color $(3, 8)$ plane, i.e., $(\phi_3, \phi_8) = \{(\varphi, 0)_1, (-\varphi, 0)_2, (\frac{\varphi}{2}, \frac{\sqrt{3}}{2}\varphi)_3, (-\frac{\varphi}{2}, -\frac{\sqrt{3}}{2}\varphi)_4, (\frac{\varphi}{2}, -\frac{\sqrt{3}}{2}\varphi)_5, (-\frac{\varphi}{2}, \frac{\sqrt{3}}{2}\varphi)_6\}$. It is worth to note that each configuration has a different color charge although all these configurations are degenerate in energy. Let us denote the color densities in configuration (1) generally as $(n_3, n_8)_1$ even though $n_3 = 0$ in this case. Then, noting that color density transforms by each discrete gauge transformation as $\langle q^+ T_\alpha q \rangle \rightarrow \langle q^+ U^+_{ab} T_\alpha U_{ab} q \rangle$, we obtain color density in each configuration $(n_{3(i)}, n_{8(i)}) = \{(n_3, n_8)_1, (-n_3, n_8)_2, (\frac{n_3}{2} + \frac{\sqrt{3}}{2}n_8, \frac{\sqrt{3}}{2}n_3 - \frac{n_8}{2})_3, (-\frac{n_3}{2} + \frac{\sqrt{3}}{2}n_8, -\frac{\sqrt{3}}{2}n_3 - \frac{n_8}{2})_4, (\frac{n_3}{2} - \frac{\sqrt{3}}{2}n_8, -\frac{\sqrt{3}}{2}n_3 - \frac{n_8}{2})_5, (-\frac{n_3}{2} - \frac{\sqrt{3}}{2}n_8, \frac{\sqrt{3}}{2}n_3 - \frac{n_8}{2})_6\}$. We notice that $n^2_{3(i)} + n^2_{8(i)} = n^2_3 + n^2_8$ is a gauge invariant quantity as it should [36].

One could argue that the average of color densities over all these configuration, $\frac{1}{6}\sum_{i=1}^{6}(n_3, n_8)_i = (0, 0)$ so that we would not need to worry about the color density. Of course this is not the case. The reason is that the thermodynamic potential of the system is not the average of those, that is $\frac{1}{6}\sum_{i=1}^{6}\Omega_{(i)}$ where $\Omega_{(i)}$ is the thermodynamic potential for the configuration $(\phi_3, \phi_8)_i$, i.e., $\beta V \Omega_{(i)} = \mathrm{Re}\, S_\varphi|_{(\phi_3, \phi_8)_i}$. To see this more clearly, we write the mean-field approximated partition function as

$$Z^{\mathrm{MF}}_{\mathrm{PNJL}}[\Phi_0, \sigma_0; \mu, T] \equiv e^{-\beta V \Omega^{\mathrm{MF}}_{\mathrm{PNJL}}[\Phi_0, \sigma_0; \mu, T]} = \sum_{(\phi_3, \phi_8)_i} e^{-\beta V \Omega_{(i)}}, \tag{20}$$

thus we have

$$\Omega^{\mathrm{MF}}_{\mathrm{PNJL}}[\Phi_0, \sigma_0; \mu, T] = -\frac{T}{V}\ln\left(\sum_i e^{-\beta V \Omega_{(i)}}\right) \overset{V \rightarrow \infty}{\longrightarrow} \min\left(\Omega_{(1)}, \Omega_{(2)}, \cdots, \Omega_{(6)}\right). \tag{21}$$

Of course, in current case with no diquark condensate, each potential is degenerate in energy although each has a different color charge. Nevertheless there is no valid argument to choose the average of $\{\Omega_{(i)}\}$ as a definition of $\Omega^{\mathrm{MF}}_{\mathrm{PNJL}}$. Furthermore, since the color symmetry is realized as a global symmetry in the PNJL model, these degenerated vacua are extremely sensitive to the infinitesimal external field which tends to align the vacuum to a specific direction, that is a famous spontaneous symmetry breakdown [9]. This can be easily illustrated as follows. If we introduce an infinitesimal external field ϵ_1 which tries to align the vacuum to $\Omega^{(1)}$ out of six directions, then we have the vacuum alignment to the $\Omega^{(1)}$ direction when the thermodynamic limit is taken as $V \rightarrow \infty$. Then after we switch off the external field $\epsilon_1 \rightarrow 0$, the vacuum remains in the (1) direction. That is

$$\Omega^{\mathrm{MF}}_{\mathrm{PNJL}}[\Phi_0, \sigma_0; \mu, T] = -\lim_{\epsilon_1 \rightarrow 0}\lim_{V \rightarrow \infty}\frac{T}{V}\ln\left(\sum_i e^{-\beta V \Omega_{(i)}}\right) = \Omega_{(1)}. \tag{22}$$
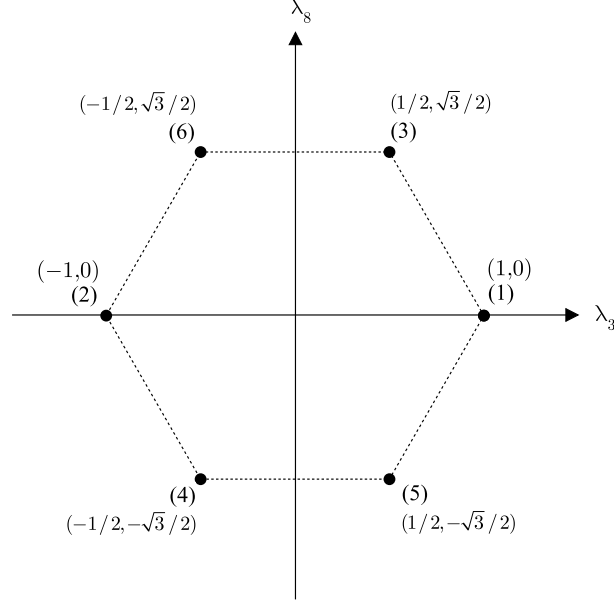
FIG. 1: The discrete gauge configurations of the Wilson line $L(\phi_3, \phi_8) = e^{i\phi_3\lambda_3 + i\phi_8\lambda_8}$ in the (λ_3, λ_8) plane which result in the same value of $\Phi = \frac{1}{3}\mathrm{tr}_c L$. The axis scale is normalized by φ such that $\Phi = (1 + 2\cos\varphi)/3$. Each configuration yields a different color charge density; Denoting the color charge density of configuration (1), as (n_3, n_8), then we have $(-n_3, n_8)$ for configuration (2), $(\frac{n_3}{2} + \frac{\sqrt{3}n_8}{2}, \frac{\sqrt{3}n_3}{2} - \frac{n_8}{2})$ for (3), $(-\frac{n_3}{2} + \frac{\sqrt{3}n_8}{2}, -\frac{\sqrt{3}n_3}{2} - \frac{n_8}{2})$ for (4), $(\frac{n_3}{2} - \frac{\sqrt{3}n_8}{2}, -\frac{\sqrt{3}n_3}{2} - \frac{n_8}{2})$ for (5), $(-\frac{n_3}{2} - \frac{\sqrt{3}n_8}{2}, \frac{\sqrt{3}n_3}{2} - \frac{n_8}{2})$ for (6). The value of $n_3^2 + n_8^2$ is the same for all configurations, and thus is gauge invariant.

In this way, the global color symmetry can be broken in the adopted mean field PNJL model, and as a consequence, nonvanishing color charge remains in the system. Whichever gauge is chosen among six configurations, the statement that color charge does not vanish is gauge invariant since $\sqrt{n_3^2 + n_8^2}$ is gauge invariant. Of course, we think that this global color symmetry breaking in the PNJL model is an artifact of the model, possibly of the adopted mean field approximation.

One may still think that this kind of problem does not appear if we take as a dynamical variable Φ itself rather than the gauge field $\{\phi_3, \phi_8\}$. In this case one can not compute each color density, so we cannot judge whether the system is color neutral or not. Furthermore this does not work as soon as we take the diquark condensation into account. In that case, we need to treat $\{\phi_3, \phi_8\}$ as the dynamical variable since the quark part of potential is no longer function of a gauge invariant quantities, Φ and $\bar{\Phi}$. Moreover, colored diquarks work as external fields which prefer some specific direction of vacuum among six configurations. When the critical temperature is approached, this external field vanishes rapidly, $\Delta \to 0$ as $T - T_c \to 0^-$, but the vacuum alignment remains in the system. In this way, we can not avoid the problem of color symmetry breaking at finite density, $\mu \neq 0$.

Before going on let us make a clarification regarding the breaking of color symmetry in the PNJL model. It is well-known that the a local gauge symmetry can never be broken as stated in the Elitzur's theorem [42]. Thus the expectation value of an operator which is not invariant under the gauge transformation should vanish after all the configurations of gauge fields are properly taken into account into the partition function. Nevertheless, it is sometimes useful to fix a gauge and define a gauge dependent order parameter in weak coupling perturbative calculations just to extract gauge independent physical quantities, as done in the BCS theory of superconductivity. In the BCS theory, one assumes a non-vanishing electron pair condensate which leads to a physical gap in a electron spectrum.

This is what happens in the PNJL model, as well as in the NJL model applied to color superconductive phases (see next Section). In the PNJL model within the adopted mean field approximation, one usually introduces a gauge dependent matrix-valued order parameter L (working in the Polyakov gauge), and then he tries to look at the gauge-independent Polyakov-loop $\Phi = \frac{1}{3}\mathrm{tr}_c L$ which is, in a pure gauge limit, related to the free energy of a single static quark excitation in a gluonic heatbath. A color-symmetry breaking in a PNJL model might be understood in this way. But this scenario immediately brings the trouble we have encountered in this section, that the gauge-independent squared sum of color densities is nonvanishing at $\mu \neq 0$ unless either $\Phi \neq 1$ or $T = 0$. After all, we conclude that this color-symmetry breaking in a PNJL model is an artifact of too naive saddle-point approximation for the gauge dependent quantity L. In order to overcome this problem, we need to somehow construct the gauge-independent approximation scheme.

IV. COLOR SUPERCONDUCTIVITY WITHIN THE PNJL MODEL

Let us finally discuss some difficulty to tackle the problem of color superconductivity in the PNJL model, although this problem is studied to some extent but with an *implicit* assumption which makes the problem much simpler [27, 37, 38]. We wish here to clarify what is the problem behind these studies, as we already mentioned in [27]. Instead of Eq. (2), we start with a matrix model combined with the NJL model described by the following Euclidean lagrangian, which would lead the system to be in the 2SC superconducting state [39, 40] at high density.

$$\mathcal{L}_E(q, \bar{q}, A_4) = q^+ (\gamma_0(\partial_\tau + iA_4) + i\gamma \cdot \nabla - \not{\mu})q - \frac{G_D}{4} \left| q^t C \gamma_5 \varepsilon_a^{(c)} \varepsilon^{(f)} q \right|^2. \tag{23}$$

We introduced $\varepsilon_a^{(c)}$ and $\varepsilon^{(c)}$, the antisymmetric matrix in color and flavor space respectively, i.e., $[\varepsilon_a^{(c)}]_{bc} = \varepsilon_{abc}$ and $[\varepsilon^{(f)}]_{ij} = \epsilon_{ij}$. Introducing the bosonic field $\Delta_a \sim \frac{G_d}{2} q^t C \gamma_5 \varepsilon_a^{(c)} \varepsilon^{(f)} q$, and integrating out fermions results in the expression of the partition function in terms of a *global* matrix model

$$Z[\mu, T] = \int_{\text{P.B.}} \Pi_x \Pi_{a=1}^3 d\Delta_a(x) d\bar{\Delta}_a(x) e^{-S_{\text{eff}}[\Delta_a(x), \bar{\Delta}_a(x)]}$$
$$e^{-S_{\text{eff}}[\Delta_a, \bar{\Delta}_a]} = \int dL e^{-\sum_a \int d\tau d\mathbf{x} \frac{\Delta_a^*(x) \Delta_a(x)}{G_D} + \frac{1}{2} \log \text{Det} \left[\begin{pmatrix} i\gamma^0 \not{D}[\mu, A_4] & -\Delta_a(x)\gamma_5 \varepsilon_a^{(c)} \varepsilon^{(f)} \\ -\Delta_a(x)\gamma_5 \varepsilon_a^{(c)} \varepsilon^{(f)} & i\not{D}[-\mu, -A_4]^t \gamma^0 \end{pmatrix} \delta_{\text{P}}(x-y) \right]}, \tag{24}$$

where the covariant derivative in the Euclidean space is defined by $i\not{D}[\mu, A_4] = -\gamma^0 \partial_\tau + i\gamma^i \partial_i + (\mu - iA_4)\gamma^0$. Here $A_4 = \sum_{\alpha=1}^8 \phi_\alpha \lambda_\alpha$ is the temporal background gauge field related to the Wilson line by $L = e^{i\beta A_4}$, $\delta_{\text{P}}(x-y)$ is again the delta function periodic in τ. The transpose operation ^{t} should be supposed to work only in color and flavor space, not in the Dirac space.

We adopt the saddle point approximation for S_{eff} and make the following homogeneous ansatz for the superconducting condensate, $\Delta_a(x) = \Delta \delta_{a3}$ with Δ being real, obtaining a matrix model inspired PNJL action

$$e^{-S_{\text{PNJL}}[\vec{\Delta}^* \cdot \Delta]} = \int dL e^{-S_\varphi[\vec{\Delta}, A_4; \mu, T]}, \tag{25}$$

where $S_\varphi = \beta V \Omega[\vec{\Delta}, A_4; \mu, T]$ whose expression will be given below, and we have introduced the condensate vector $\vec{\Delta} = (0, 0, \Delta)$ in the color antitriplet space. We have taken care the fact that when integrated over L, the action S_{eff} is only a function of the gauge invariant convolution $\vec{\Delta}^* \cdot \Delta$, while it is not before the integration, i.e., S_φ, or equivalently Ω depend on the relative angle of L and $\vec{\Delta}$ in the color space as we shall see later. The real part of the thermodynamic potential in the presence of a constant background gauge A_4 can be calculated by [3]

$$\text{Re}\,\Omega[\vec{\Delta}, A_4; \mu, T] = \frac{\Delta^2}{G_D} - \sum_{A=1}^{24} \int \frac{d\mathbf{p}}{(2\pi)^3} \left[\text{Re}\, E_A(p) + 2T \ln \| 1 + e^{-E_A(p)/T} \| \right]. \tag{26}$$

$E_A(p)$ is chosen from the eigenvalues of the following non-hermitian 48 by 48 hamiltonian density matrix such that $\text{Re}\, E_A$ is positive.[4] The index A refers to the quasiquark degrees of freedom that can be enumerated as 3(color) × 2(flavor) × 4(Dirac).

$$\mathcal{H}(p) = \begin{pmatrix} \alpha \cdot \mathbf{p} - (\mu - iA_4) & \Delta \gamma_5 \varepsilon_3^{(c)} \varepsilon^{(f)} \\ \Delta \gamma_5 \varepsilon_3^{(c)} \varepsilon^{(f)} & \alpha \cdot \mathbf{p} + (\mu - iA_4) \end{pmatrix}, \tag{27}$$

where $\alpha^i = \gamma^0 \gamma^i$. It is important to note now that the direction of color symmetry breaking is gauge fixed to the third direction in color space.

With this fact in mind, let us see what happens when we wish to integrate in the SU(3) matrix L. We again make use of the polar/phase decomposition of L as

$$L = g^{-1} L_{\text{diag}} g, \quad dL = \mu(\varphi_1, \varphi_2) d\varphi_1 d\varphi_2 [dg]. \tag{28}$$

---

[3] The readers are refereed to [27, 41] for more detail of the derivation.
[4] This is always possible since the eigenvalues of the Hamiltonian density appears in sets of eigenvalues, $\{E_A, -E_A\}$ because of the Nambu-Gorkov degeneracy.

Ignoring the imaginary part of the PNJL model action as usual, the partition function becomes

$$
\begin{aligned}
Z_{\text{PNJL}}(\mu, T) &= \int \mu(\varphi_1, \varphi_2) d\varphi_1 d\varphi_2 [dg] e^{-\text{Re}\, S_\varphi[\vec{\Delta},\, g^{-1} A_4^{\text{diag}} g;\, \mu, T]} \\
&= \int \mu(\varphi_1, \varphi_2) d\varphi_1 d\varphi_2 [dg] e^{-\text{Re}\, S_\varphi[g^*\vec{\Delta},\, A_4^{\text{diag}};\, \mu, T]},
\end{aligned}
\tag{29}
$$

where g^* is the complex conjugate of the phase matrix g. Here we clearly see that the phase g does not disappear from the integrand so that we can not trivially perform the integration over the phase. This means that the saddle-point solutions to Z_{PNJL} with respect to the gauge fields $\{\varphi_1, \varphi_2\}$ clearly depend on the relative angle g^* between A_4 and $\vec{\Delta}$. This is in contrast to the case where we do not have the diquark condensate in the system. This means that we can no longer restrict ourselves to diagonal form of the temporal gauge A_4 when the orientation of diquark condensate is fixed to the third axis of the color space. Otherwise, we can not limit ourselves to the assumption $\vec{\Delta} = (0, 0, \Delta)$ when A_4 is parametrized by a diagonal form. This fact is already stressed in [27], but it is usually assumed in the literatures that both A_4 is diagonal and $\Delta = (0, 0, \Delta)$ [37].

What happens in that case? Let us assume without justification that the integration over $[dg]$ can be done

$$
Z_{\text{PNJL}}(\mu, T) = \int \mu(\varphi_1, \varphi_2) d\varphi_1 d\varphi_2 e^{-\beta V \text{Re}\, \Omega[\vec{\Delta}, A_4^{\text{diag}};\, \mu, T]},
\tag{30}
$$

where $\vec{\Delta} = (0, 0, \Delta)$. Then we work the saddle point approximation in φ_1 and φ_2. Again restricting Φ to be real, letting $\Phi = \frac{1 + 2\cos\varphi}{3}$, the partition function becomes

$$
Z_{\text{PNJL}}^{\text{MF}}(\mu, T) = \sum_{\{\phi_3, \phi_8\}_i} e^{-\beta V \left[\text{Re}\, \Omega[\vec{\Delta}, A_{4(i)}^{\text{diag}};\, \mu, T] + V_{\text{glue}}[\Phi, \Phi] \right]}.
\tag{31}
$$

$A_{4(i)}^{\text{diag}}$ is the temporal background gauge field in the configuration $\{\phi_3, \phi_8\}_i$. We have exponentiated the Vandermonde determinant and introduced a phenomenological Polyakov loop potential as before. In this case, the degeneracy in six discrete gauge configurations is lifted away. We have two degenerated minima in this case, that is configuration (1) and (2). This is a kind of vacuum alignment, because only the configuration (1) and (2) do not bring any additional symmetry breaking in $\text{Re}\,\Omega$ but the original one, i.e., $\text{SU}(3)_c \to \text{SU}(2)_c$ caused by diquark condensate $\vec{\Delta} = (0, 0, \Delta)$. In the thermodynamic limit $V \to \infty$, the state should be selected to be either in configuration (1) or (2) again not to their average. Even though both states in this case turn out to have the same color charge density $(0, n_8)$ because of vanishing n_3, the physical state cannot be the statistical (symmetric) average of these two states because the PNJL model has only a global color symmetry, and the quantum field theory has an infinite number of degrees of freedom in the limit $V \to \infty$. This is in contrast to quantum mechanics where we can construct the symmetric states by average of two states. When temperature is increased, the 2SC state eventually undergoes a 2nd order transition to the normal quark matter phase. Δ decreases to zero when the critical temperature is approached, but the alignment of A_4 in the color space remains in (1) or (2) configuration, both of which yield nonvanishing color density $n_8 \neq 0$ even in the normal quark matter phase at $\mu \neq 0$. We thus conclude that the PNJL model without color chemical potentials at finite $\mu \neq 0$ is not free from appearance of color charge density even if diquark fields are absent.

V.   CONCLUSIONS

We have discussed how color neutrality should be implemented in the adopted mean field PNJL model at finite density. We have given detailed formal arguments that explain why we need to introduce a color chemical potential at $\mu \neq 0$ in order to assure color neutrality of the system. We have compared our results with those obtained in Ref. [28], showing that there is not discrepancy at $\mu = 0$ (which is the regime investigated numerically in the aforementioned paper).

Moreover we have pointed out that the mean field solution can not be computed as an average over the degenerate ground states: as a matter of fact, spontaneous symmetry breaking means that one can choose only one of the degenerate ground states. In fact, the various ground states of the PNJL model can not communicate among them because they construct different Hilbert/Fock spaces, i.e., the overlap between those spaces vanishes after $V \to \infty$; we should choose only one of them for making calculations. This procedure, which is common both to discrete and to continuous symmetry, is very popular in statistical mechanics, when one for example studies effective models of spontaneous magnetization of Weiss domains: Nature chooses only one direction for magnetization, even if the ground state possesses an infinity of degenerate ground states corresponding to the infinite possible orientation of the magnetization.

We have also discussed a subtle point related to the simultaneous diagonalization of the Polyakov loop L and the color superconductive gap matrix Δ. We have shown that in principle it is not correct to take both of them in a

diagonal form, since one needs two color rotations in order to achieve this: the color rotation that makes L diagonal does not necessarily make Δ diagonal and vice versa. Thus in principle one should work either with a diagonal L or with a diagonal Δ. Furthermore we have noticed that, defining a non superconductive state as a limit for $\Delta \to 0$ of a superconductive one, the vacuum alignment makes both mathematically and physically impossible to define the ground state as a mean of the several degenerate ground states of the model. As a consequence, we can not avoid non-vanishing color charge density even in the non superconducting phase without introducing appropriate color chemical potentials, at least in the mean field approximation adopted here.

[1] Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, Phys. Lett. B **643**, 46 (2006) [arXiv:hep-lat/0609068].
[2] C. Schmidt, PoS **LAT2006**, 021 (2006) [arXiv:hep-lat/0610116].
[3] O. Philipsen, PoS **LAT2005**, 016 (2006) [PoS **JHW2005**, 012 (2006)] [arXiv:hep-lat/0510077].
[4] U. M. Heller, PoS **LAT2006**, 011 (2006) [arXiv:hep-lat/0610114].
[5] J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984).
[6] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 465 (1985).
[7] P. A. M. Guichon, Phys. Lett. B **200**, 235 (1988).
[8] K. Saito and A. W. Thomas, Phys. Lett. B **327**, 9 (1994) [arXiv:nucl-th/9403015].
[9] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[10] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961).
[11] S. Ejiri, Phys. Rev. D **69**, 094506 (2004) [arXiv:hep-lat/0401012].
[12] K. Splittorff, PoS **LAT2006**, 023 (2006) [arXiv:hep-lat/0610072].
[13] K. Splittorff and J. J. M. Verbaarschot, Phys. Rev. Lett. **98**, 031601 (2007) [arXiv:hep-lat/0609076].
[14] O. Nachtmann and W. Wetzel, Phys. Lett. B **81**, 211 (1979).
[15] T. Banks and A. Ukawa, Nucl. Phys. B **225**, 145 (1983) .
[16] F. Karsch and E. Laermann, Phys. Rev. D **50**, 6954 (1994) [arXiv:hep-lat/9406008].
[17] S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. D **57**, 3910 (1998) [arXiv:hep-lat/9710048].
[18] A. M. Polyakov, Phys. Lett. B **72**, 477 (1978).
[19] L. Susskind, Phys. Rev. D **20**, 2610 (1979).
[20] B. Svetitsky and L. G. Yaffe, Nucl. Phys. B **210**, 423 (1982).
[21] S. P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992).
[22] T. Hatsuda and T. Kunihiro, Phys. Rept. **247**, 221 (1994) [arXiv:hep-ph/9401310].
[23] M. Buballa, Phys. Rept. **407**, 205 (2005) [arXiv:hep-ph/0402234].
[24] J. Bardeen, L. N. Cooper and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[25] K. Fukushima, Phys. Lett. B **591**, 277 (2004) [arXiv:hep-ph/0310121].
[26] P. N. Meisinger and M. C. Ogilvie, Phys. Lett. B **379**, 163 (1996) [arXiv:hep-lat/9512011].
[27] H. Abuki, M. Ciminale, R. Gatto, G. Nardulli and M. Ruggieri, Phys. Rev. D **77**, 074018 (2008) [arXiv:0802.2396 [hep-ph]]; H. Abuki, arXiv:0805.3076 [hep-ph].
[28] T. Hell, S. Roessner, M. Cristoforetti and W. Weise, arXiv:0810.1099 [hep-ph].
[29] S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa and R. Ray, Phys. Rev. D **77**, 094024 (2008) [arXiv:0710.2790 [hep-ph]].
[30] A. Dumitru, R. D. Pisarski and D. Zschiesche, Phys. Rev. D **72**, 065008 (2005) [arXiv:hep-ph/0505256].
[31] H. Abuki, M. Ciminale, R. Gatto, N. D. Ippolito, G. Nardulli and M. Ruggieri, Phys. Rev. D **78**, 014002 (2008) [arXiv:0801.4254 [hep-ph]].
[32] H. Abuki, R. Anglani, R. Gatto, G. Nardulli and M. Ruggieri, arXiv:0805.1509 [hep-ph].
[33] H. Abuki, R. Anglani, R. Gatto, M. Pellicoro and M. Ruggieri, arXiv:0809.2658 [hep-ph].
[34] J. B. Kogut, M. Snow and M. Stone, Nucl. Phys. B **200**, 211 (1982).
[35] D. Gomez Dumm, D. B. Blaschke, A. G. Grunfeld and N. N. Scoccola, arXiv:0807.1660 [hep-ph].
[36] M. Buballa and I. A. Shovkovy, Phys. Rev. D **72**, 097501 (2005) [arXiv:hep-ph/0508197].
[37] S. Roessner, C. Ratti and W. Weise, Phys. Rev. D **75**, 034007 (2007) [arXiv:hep-ph/0609281].
[38] M. Ciminale, R. Gatto, N. D. Ippolito, G. Nardulli and M. Ruggieri, Phys. Rev. D **77**, 054023 (2008) [arXiv:0711.3397 [hep-ph]].
[39] R. Rapp, T. Schafer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998) [arXiv:hep-ph/9711396].
[40] M. G. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B **422**, 247 (1998) [arXiv:hep-ph/9711395].
[41] H. Abuki and T. Kunihiro, Nucl. Phys. A **768**, 118 (2006) [arXiv:hep-ph/0509172].
[42] S. Elitzur, Phys. Rev. D **12**, 3978 (1975).